Oxford Cambridge and RSA

Friday 24 May 2024 – Afternoon

AS Level Further Mathematics B (MEI)

Y413/01 Modelling with Algorithms

Time allowed: 1 hour 15 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

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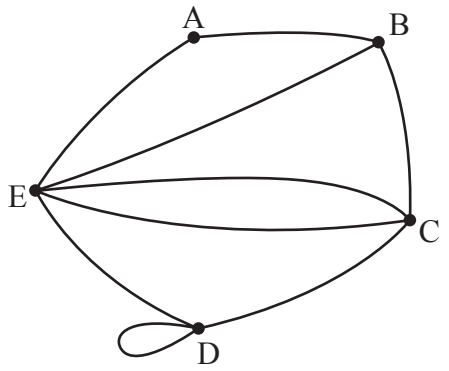
1 **Fig. 1** shows a graph.

Fig. 1



(a) **(i)** State the order of node B. **[1]**

(ii) State the order of node D. **[1]**

(b) Complete the incidence matrix for the graph in **Fig. 1** in the Printed Answer Booklet. **[1]**

(c) Explain why the graph in **Fig. 1** is not simple. **[1]**

(d) Write down the number of arcs that need to be removed from the graph in **Fig. 1** for it to be a tree. **[1]**

2 The list below shows the sizes of nine items.

31 13 12 17 25 18 11 8 15

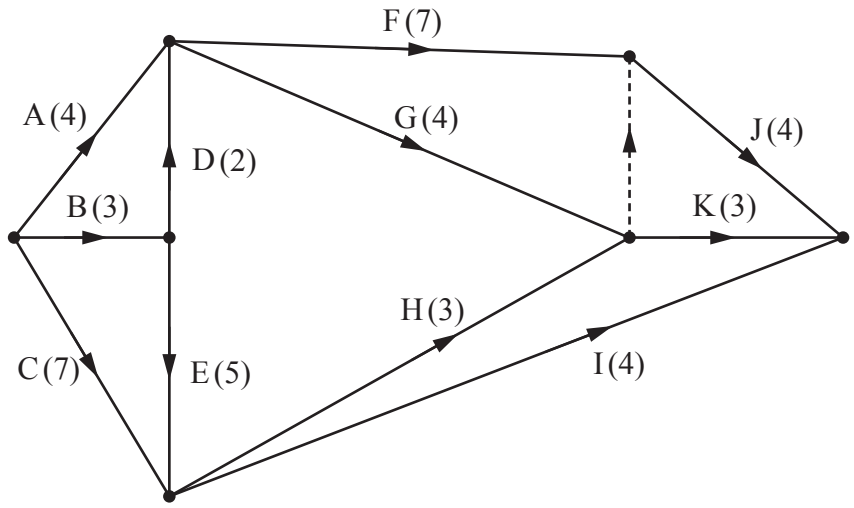
(a) Show the result of applying the first fit algorithm to pack items with the sizes listed above into bins that have a capacity of 50. **[1]**

(b) Apply the quick sort algorithm to sort the list of numbers above into **ascending** order. You should use the **first** value as the pivot for each sublist. **[3]**

(c) Show the result of applying the first fit decreasing algorithm to pack items with the sizes listed above into bins that have a capacity of 50. **[2]**

(d) By finding an optimal packing using a full bin strategy, explain why both first fit and first fit decreasing are examples of heuristic algorithms. **[2]**

3 The diagram shows an activity network for a project. The number in brackets show the duration of these activities in hours.



(a) Complete the table in the Printed Answer Booklet to show the immediate predecessors for each activity. **[2]**

(b) **(i)** Carry out a forward pass and a backward pass through the activity network, showing the early event time and the late event time at each vertex of the network. **[3]**

(ii) State the minimum project completion time. **[1]**

(iii) State the critical activities of the project. **[1]**

(c) Calculate the independent float for activity I. **[1]**

Each activity requires one person. When an activity is started it must be completed without interruption.

(d) Use the diagram in the Printed Answer Booklet to show how **three** people can complete the project in the minimum time found in **3(b)(ii)**. Each column in the diagram represents 1 hour. For each person, write the letter of the activity they are doing in each box, or leave the box blank if the person is resting for that 1 hour. **[2]**

4 A student sets up the following initial tableau in **Fig. 4.1** to solve a maximisation LP problem in x, y, and z.

Fig. 4.1

P	x	y	z	s_1	s_2	s_3	RHS
1	-2	3	-1	0	0	0	0
0	1	2	3	1	0	0	k
0	3	-1	1	0	1	0	45
0	2	4	-1	0	0	1	50

(a) Formulate the information from **Fig. 4.1** as a linear programming problem, stating the objective and listing all the constraints as simplified inequalities with integer coefficients. **[3]**

It is given that k is a constant where $15 < k < 135$.

(b) Perform **one** iteration of the simplex method, using an entry in the x column as the pivot element. Give each entry as an exact value. **[3]**

The tableau after a second iteration of the simplex method, is given in **Fig. 4.2**.

Fig. 4.2

P	x	y	z	s_1	s_2	s_3	RHS
1	0	$\frac{21}{8}$	0	$\frac{1}{8}$	$\frac{5}{8}$	0	$\frac{k+225}{8}$
0	0	$\frac{7}{8}$	1	$\frac{3}{8}$	$-\frac{1}{8}$	0	$\frac{3(k-15)}{8}$
0	1	$-\frac{5}{8}$	0	$-\frac{1}{8}$	$\frac{3}{8}$	0	$\frac{-k+135}{8}$
0	0	$\frac{49}{8}$	0	$\frac{5}{8}$	$-\frac{7}{8}$	1	$\frac{5(k+17)}{8}$

(c) Explain how the tableau in **Fig. 4.2** shows that the solution obtained after a second iteration is optimal. **[1]**

(d) Given that the optimal value of P is 36, determine the corresponding value of each basic variable after the second iteration. **[3]**

5 The objective function for a LP formulation to find the shortest path from A to G in a network with seven vertices, A, B,…, G is given below.

Minimise 5AB + 13AC + 8AD + 6BC + 6CB + 14BF + 14FB + 12BE + 12EB + 6DE + 6ED + 6CF + 6FC + 2EF + 2FE + 22DG + 11EG + 7FG

(a) Explain why both BC and CB appear in the objective function. **[1]**

One of the constraints of the LP formulation is

AD + ED − DE − DG = 0

(b) Explain the purpose of this constraint in the LP formulation. **[1]**

(c) Using the vertices given in the Printed Answer Booklet draw a network with **12** arcs to represent the information given in the above objective function. **[2]**

(d) **(i)** Apply Dijkstra's algorithm to the network drawn in part **(c)**, to find the weight of the shortest path from A to G. **[5]**

(ii) Write down the shortest path from A to G. **[1]**

(e) A computer takes 0.014 seconds to solve a shortest path problem on a network with seven vertices using Dijkstra's algorithm. Approximately how long, in seconds, will it take the same computer to solve a shortest path problem on a network with seven hundred vertices using Dijkstra's algorithm? **[2]**

(f) **(i)** Apply Prim's algorithm, starting at A, to find the minimum spanning tree for the network drawn in part **(c)**. You must, as part of your solution, state the order in which the arcs were included in the tree. **[2]**

(ii) State the weight of the minimum spanning tree. **[1]**

6 Each year, to raise money for charity, Henry makes and sells three different sizes of T-shirt, small, medium, and large.

This year, the number of T-shirts that Henry plans to make are subject to the following constraints.

Each small T-shirt requires 25 minutes to make, each medium T-shirt requires 40 minutes to make, and each large T-shirt requires 60 minutes to make.

The total time that Henry can spend making all the T-shirts must not exceed 100 hours.

From his experience in previous years, Henry decides to make at most 50 medium T-shirts and at least 100 T-shirts in total.

Furthermore, Henry decides to make small and large T-shirts in the ratio of 2 small T-shirts for every 5 large T-shirts.

Henry plans on selling each small T-shirt for £5, each medium T-shirt for £6 and each large T-shirt for £8. Henry wants to maximise the total income from the sale of the T-shirts. Total income is the money made from selling T-shirts, ignoring the cost of materials.

Let x, y and z represent the number of small T-shirts, medium T-shirts, and large T-shirts respectively that Henry makes.

(a) By representing the feasible region for x and y graphically, determine how many of each size of T-shirt Henry should make to maximise the total income from selling the T-shirts. You must show all your working. **[10]**

(b) Using the solution found in part **(a)**, find the maximum possible total income. **[1]**

(c) Give a reason why the total income found in part **(b)** may not be achieved. **[1]**

END OF QUESTION PAPER

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